Signature of the electron-electron interaction in the magnetic field dependence of nonlinear I-V characteristics in mesoscopic systems.

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We show that the nonlinear I-V characteristics of mesoscopic samples with metallic conductivity should contain parts which are linear in the magnetic field and quadratic in the electric field. These contributions to the current are entirely due to the electron-electron interaction and consequently they are proportional to the electron-electron interaction constant. We also note that both the amplitude and the sign of the current exhibit random oscillations as a function of temperature.

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According to the Onsager relation the linear conductance $G(\mathbf{H})$ of a conductor measured by the two-probe method must be an even function of the magnetic field $\mathbf{H}$ [1]:

$$G(\mathbf{H}) = G(-\mathbf{H}) \tag{1}$$

Eq.1 is a consequence of general principles: the time reversal symmetry and the positive sign of the entropy production. Therefore it holds in all conductors. It is possible however that that nonlinear I-V characteristics of conductors contain parts odd in $\mathbf{H}$. In particular, one can have contributions to the total current through a sample which are linear in H and quadratic in the voltage across the sample V.

$$I_{(nl)} = \alpha V^2 H \tag{2}$$

Since $\mathbf{H}$ is an axial vector and the current is a polar one, the coefficient α can be non-zero only in non-centrosymmetric media. In the case of bulk non-centrosymmetric crystals terms in $I-V$ characteristics that are linear in $\mathbf{H}$ have been investigated both theoretically, the using Boltzmann kinetic equation, and experimentally (See for example [2]). In the case of chiral carbon nanotubes a classical theory of this effect was discussed in [3].

In this article we study this effect at small temperatures and in mesoscopic disordered samples where all possible symmetries are broken. In this situation all electron transport effects are of a quantum interference nature. The theory of nonlinear characteristics of mesoscopic metallic samples was developed in the approximation of non-interacting electrons [4, 6]. It is important, however, that in this approximation $\alpha = 0$ and magnetic field dependence of the $I-V$ characteristics is an even function of $\mathbf{H}$. Therefore the coefficient α in Eq.2 should be proportional to electron-electron interaction constant β, which is defined by the interacting part of the electron Hamiltonian

$$H_{(int)} = \frac{\beta}{\nu}\int d\mathbf{r}\Psi(\mathbf{r})\Psi^*(\mathbf{r})\Psi(\mathbf{r})\Psi^*(\mathbf{r}) \tag{3}$$

Here ν is the electron density of states. Thus, in principle, by measuring the current in Eq.2 one can measure the electron-electron interaction constant β.

Let us consider a sample of two-dimensional geometry shown in the insert of Fig.1 and assume that the magnetic field is perpendicular to the plane and that the characteristic size of the sample $L \gg l$ is much larger then the electron elastic mean free path l. At low temperatures the main contributions to both mesoscopic fluctuations of the conductance $\delta G = G - \langle G\rangle$ and the nonlinear current Eq.2 are due to electron interference effects. As usual in such situations α is random sample specific quantities with zero average $\langle\alpha\rangle = 0$. To characterize α one has to calculate the variance $\langle\alpha^2\rangle$. Here the brackets $\langle\rangle$ denote averaging over realizations of a random white noise scattering potential characterized by a correlation function $\langle u(\mathbf{r})u(\mathbf{r})\rangle = \frac{\pi}{lm^2}\delta(\mathbf{r}-\mathbf{r}')$, where m is the electron mass.

A diagrammatic verification of Eq.1 in the situation when interference corrections to conductance are significant is not entirely trivial and involves calculation of diagrams shown in Fig.1a,b [7]. We will use a standard diagram technique for averaging over random realizations of the scattering potential [5]. To verify that the correlation function of conductances $\langle\delta G(\mathbf{H})\delta G(0)\rangle$ is an even function of the magnetic field one has to take into account contributions to the correlation function of "Diffuson" and "Cooperon" propagators which are represented by the ladder diagrams in Fig.1 a,b. (The "Diffuson" is the ladder part of the diagram in Fig.1a where the direction of arrows of the electron Green function is antiparallel, while the "Cooperon" is the ladder in Fig.1b with parallel directions of the arrows. They contain parts linear in the magnetic field, which are equal in magnitude and of different signs. Thus these contributions cancel. To verify the fact that $\alpha = 0$ in the approximation of non-interacting electrons one has to calculate diagrams shown in Fig.1c. In this case the linear in $\mathbf{H}$ the Diffuson and the Cooperon contributions will cancel each other as well. This fact is also quite obvious in the framework of

Landauer scheme of calculations of the conductance.

In linear in β approximation the variance

$$\langle\alpha^2\rangle = \beta^2 \frac{e^2}{\nu^2\Gamma^4 A^2}(\frac{e^2}{h})^2(\frac{L^2}{\Phi_0})^2 \tag{4}$$

is given by diagrams shown in Fig.1d. Here $\Gamma = \hbar D/L^2$ and D/L^2 is the inverse lifetime of an electron in the sample, A is the area of the sample, and $D = v_F l/2$ is the electron diffusion coefficient. Eq. 4 is valid at $eV \ll \Gamma$ and $\Phi \ll \Phi_0$, where $\Phi = HA$ and Φ_0 are the magnetic flux through the sample area and the flux quanta respectively. For simplicity we consider the short range e-e interaction described by Eq.3. In this case the Hartree term is twice larger than the exchange term and we consider only the diagram shown in Fig.1d.

In the case of the Coulomb interaction between electrons and at high electron densities we have

$$\beta = e^2 r_D \nu \tag{5}$$

where r_D is the screening radius ($e^2 r_D/D \sim 1/k_F l$).

The existence of the effect described by Eq.4 is connected to the fact that in the presence of a voltage V across the sample there is a part of the local density

$$\delta n(\mathbf{r}) \sim VH \tag{6}$$

which is proportional to H and V [8]. Existence of fluctuations of the density of the form Eq.6 is a consequence of the fact that linear in $\mathbf{H}$ parts of the "Diffusion" and "Cooperon" digrams shown in Fig.1d do not cancel. For our estimate it is enough to consider only "Diffuson" contribution.

We would like to stress that the effect described by Eq.4 is quite different from conventional effects in bulk crystals which can be described by the Boltzmann kinetic equation [2, 3]. The leller effects are determined by relaxation processes in materials with complicated band structures and for this reason they are proportional to the the relaxation rates (or proportional to β^2), while Eq.2 is proportional to β.

The qualitative explanation of Eq.4 is the following. The mesoscopic fluctuations of the current density inside the sample are due to random interference of electron waves traveling along different diffusive paths. Though, the total current through the sample should be an even function of $\mathbf{H}$ the local current densities contain a part proportional to $\mathbf{H}$. For example, a part of the current density proportional to $\mathbf{H}$ can be characterized as a "Hall current density". To avoid confusion we would like to mention that this "Hall component" is connected with the electric field in highly non-local way and has a random direction. By the same token in random system there is a component of electron density described by Eq.6. We note also that density fluctuations Eq.6 are different from Friedel oscillations in disordered samples, which are even function of $\mathbf{H}$. In the Hartree approximation there is an additional scattering potential

$$\delta u_e(\mathbf{r}) = \frac{\beta}{2\nu}\delta n(\mathbf{r}) \tag{7}$$

associated with the fluctuations of the electron density. Thus we can write an expression for a total current in the form

$$I = G[V, T, H, \{\delta u_e(\mathbf{r}, H)\}]V \tag{8}$$

where the nonlinear conductance $G = G_D + \delta G(V, T, H, \{\delta u_e(\mathbf{r}, H)\})$, generally speaking, depends on the realization of $u_e(\mathbf{r})$ via the corresponding dependence of the mesocopic part δG. Here $G_D = e^2 D\nu$ is the Drude conductance.

The sensitivity of the sample conductance to a change in the scattering potential $\delta u_e(\mathbf{r})$ has been considered in [10, 11]. Generally speaking, the mesoscopic part of the conductance δG depends on all spatial harmonics of $\delta u_e(\mathbf{r})$. However, the main contribution to the change of the conductance comes from zero harmonics of the potential

$$\delta\bar{u}_e(V, \mathbf{H}) = \frac{\beta}{\nu A}\int(\delta n(\mathbf{r}, V, H) - \delta n(\mathbf{r}, V, 0))d\mathbf{r} \tag{9}$$

This can be verified by making calculations similar to those in [9]. This is also related to the long range character of the correlation function of the part of the electron densities, which are proportional to V [8]. For example, in the 2D case the correlation function described by digrams shown in Fig.1c has the form

$$\langle\delta u_e(\mathbf{r})\delta u_e(\mathbf{r}')\rangle = \frac{\beta^2}{\nu^2}\langle\delta n(\mathbf{r})\delta n(\mathbf{r}')\rangle = -\frac{\beta^2}{2\pi\nu^2}(\frac{V}{\pi}\,hD)^2 \ln\frac{|\mathbf{r}-\mathbf{r}'|}{L} \tag{10}$$

Expanding Eq.8 ($\delta I = (d\delta G/d\bar{u}_e)\bar{u}_e V$) in terms of $\bar{u}_e$ and taking into account that in the main approximation $\bar{u}$ and δG are uncorrelated we get

$$\langle I^2_{(nl)}\rangle = \langle(dG/d\bar{u}_e)^2\rangle\langle\bar{u}_e^2\rangle V^2 \tag{11}$$

According to [11] $\langle(dG/du)^2)\rangle = (e^2/h)^2/\Gamma^2$. Calculating the correlation function

$$\langle\delta(\bar{u}_e(H))^2\rangle = \frac{\beta^2}{\nu^2}\frac{1}{A^2}\frac{\Phi^2}{\Phi_0}\frac{|eV|^2}{\Gamma} \tag{12}$$

we arrive at Eq.4.

At this point we would like to mention that on a qualitative level the effect considered above can be also described in the framework of Landauer-Buttiker scheme. To do so one has to combine results of [12] and [13].

Eq.4 is valid at small temperatures $\Gamma \gg T$. At finite temperature the quantity $\langle\alpha^2(T)\rangle$ decreases with T. At $T \gg \Gamma$

$$\langle\alpha^2(T)\rangle \sim \langle\alpha^2(0)\rangle\frac{\Gamma^2}{T^2} \tag{13}$$

We stress that the temperature dependence of $\alpha(T)$ is non-monotonic: $\alpha(T)$ exhibits random oscillations in magnitude and sign, superimposed on the average decay. One can see this by calculating the quantity

$$\langle \alpha(T)\alpha(0)\rangle \sim \langle \alpha^2(0)\rangle \frac{\Gamma^2}{T^2} \tag{14}$$

Note that Eqs.13,14 have the same temperature dependence, which is impossible without oscillations of the sign of $\alpha(T)$ [14].

In the case of high magnetic field $\Phi > \Phi_0$, (but still $eV \ll \Gamma$), the part of the current which is the asymmetric in H and quadratic in V exhibits random oscillations as a function of Φ. These oscillations, typical for mesoscopic systems have characteristic period Φ_0 and the amplitude

$$\langle (I(\mathbf{H}) - I(-\mathbf{H}))^2\rangle = \beta^2 (\frac{eV}{\Gamma})^2 \frac{1}{(\nu\Gamma A)^2} (\frac{e^2V}{h})^2 \tag{15}$$

Finally, we mention that there are no mechanisms contributing to Eq.2 other than the mechanism considered above. For example, at finite V there is a new channel of electron transmission through the sample when an incident electron is transmitted into two electrons and a hole. The probability of such a process has a component which is linear in H. It's magnitude can be estimated in a way similar to the estimating the electron- electron scattering rate of quasiparticles in a uniform Fermi liquid. As a result, it is proportional to V^2. Thus the magnitude of the asymmetric-in-H part of the current associated with such process is proportional to V^3H.

Recently discussed above effect was observed experimentaly in $GaAs$ quantum dots [16].

During the preparation of the manuscript we became aware of similar unpublished results by D. Sanchez and M. Büttiker [15].

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[14] We note that similar phenomenon exist in the case of mesoscopic oscillations of linear conductance. Namely the quantity $\Delta G = (G(H,T) - G(0,T))$ exhibits random oscillations both in magnitude and in sign as a function of T. One can see this by calculating the variances $\langle(\Delta G(T))^2\rangle \sim \langle \Delta G(T)\Delta G(0)\rangle \sim 1/T$. Both quantities have the same T-dependence, which is impossible if sign $\Delta G(T)$ does not oscillate as a function of T. As far as we know this fact has not been discussed in the literature.
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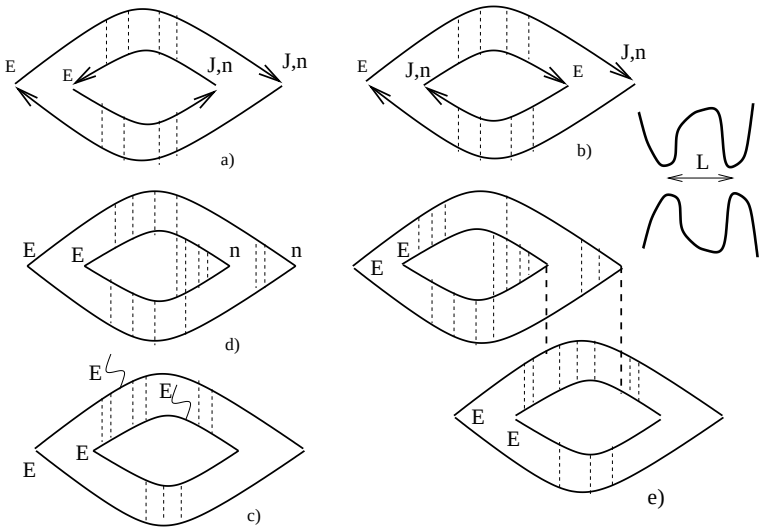

FIG. 1: Solid lines correspond to electron Green's functions, thin dashed lines correspond to the correlation function of the random scattering potential $\langle u(\mathbf{r})u(\mathbf{r}')\rangle$, thick dashed lines correspond to the electron-electron interaction β/ν. Symbols E, J, n correspond to electric field, current density and electron density respectively. Diagrams a) and b) describe the correlation function $\langle G(\mathbf{H})G(0)\rangle$. Parallel and antiparallel directions of arrows in the ladder parts of these diagrams correspond to the Cooperon and Diffusons respectively. The diagram c) describes the mesoscopic fluctuations of the part of the electron density which is proportional to the electric field squared in the approximation of noninteracting electrons. The diagram d) describes Eq.10. The diagram e) describes Eqs.4, 13, 14. The insert shows a schematic picture of the sample.